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A temperature wall function for turbulent mixed convection from vertical, parallel plate channels

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Abstract

Despite tremendous increase in computing power and an attendant increase in direct numerical simulations (DNS) of turbulent flows, near wall treatment continues to be the mainstay of most commercial packages as well as in house codes, for wall bounded turbulent flows. Even so efficient near wall treatments of truly "mixed convection flows" are not many in the open literature. This paper looks at a possible approach to the treatment of a typical mixed convection problem from vertical, parallel plate channels. The procedure employed is an asymptotic blending of natural and forced convection wall functions, which themselves are derived based on an asymptotic analysis of flow in the near wall region. Testing of the wall function was done for both aiding and opposing flows with DNS data available in literature.

Keywords: Mixed convection; Wall function; Temperature; Blending; Asymptotics; Vertical channels

1. Introduction

In many applications involving forced convection heat transfer, the effect of buoyancy is not negligible. Such flows are known as "buoyancy affected flows" or more correctly as mixed convection flows. When the Reynolds number (*Re*) or the Grashof number (*Gr*) is high enough, the flow is turbulent. Pioneering work on turbulent mixed convection particularly from vertical tubes has been done by Jackson and co-workers [1–3]. Excellent reviews on turbulent mixed convection have been done by Jackson et al. [4] and Jackson [5]. Nakajima et al. [6] studied the effect of buoyancy on the turbulent transport processes in mixed convection for both aiding and opposing flows. They conducted experiments on mixed convection between vertical parallel plates at different temperatures, for which the flow is fully developed. They proposed an analytical model, based upon the damping factor concept for combined natural and forced convection and the results were verified experimentally. Joye [7] reported the results of experiments on opposing, mixed convection heat transfer on a vertical tube and compared the results against predictions by earlier correlations and concluded that existing correlations predicted the heat transfer quite well. Kasagi and Nishimura [8] performed a direct numerical simulation (DNS) of combined forced convection and natural convection in a vertical plane channel. One wall of the channel was heated while the other was cooled and in this way one gets to study the mechanisms of fluid flow and heat transfer in both aiding and opposing flows simultaneously. Costa et al. [9] investigated, numerically and experimentally, the problem of confined, mixed convection air flow generated by two non-isothermal plane wall jets in a square enclosure. Eight low Reynolds number k–ε models, with a simplified version of the two layer wall function model of Chieng and Launder [10] were tested. Agarwal et al. [11] tested the suitability of the pseudo-compressibility algorithm for mixed convection flow problems for both laminar and turbulent flows by choosing the driven cavity problem as a test case. To tackle the problem of pressure-velocity coupling, they introduced an artificial compressibility term into the continuity equation and called it as the pseudo-compressibility approach. Chen and Chang [12]

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Nomenclature

English symbols

C	constant of the overlap temperature profile (Eq. (20))	
D	constant of the overlap temperature profile (Eq. (20))	
g	acceleration due to gravity, 9.81 m/s^2	
Gr	Grashof number, $g\beta\Delta\overline{T}H^3/\nu^2$	
Gr_t	turbulent Grashof number, $g\beta T_c H^3/\nu^2$	
H	characteristic dimension (spacing for the vertical channel)	m
Nu	Nusselt number	
Pr	Prandtl number, ν/α	
q_{turb}	turbulent heat flux	W/m^2
q^+_{turb}	dimensionless turbulent heat flux, q_{turb}/q_w	
q_w	wall heat flux	W/m^2
Ra_t	turbulent Rayleigh number, $\frac{g\beta T_c H^3}{\alpha\nu}$	
Ra	Rayleigh number, $g\beta\Delta\overline{T}H^3/\nu\alpha$	
Re	Reynolds number, $u_\infty H/\nu$	
Re_t	turbulent Reynolds number, $u_\tau H/\nu$	
Ri	Richardson number, Gr/Re^2	
Ri_t	turbulent Richardson number, Gr_t/Re_t^3	
T_c	characteristic temperature, defined in Eq. (6)	
T_C	characteristic temperature, defined in Eq. (3)	K
T_H	hot wall temperature	K
T_o	reference temperature	K
T_W	wall temperature	K
$\overline{T}$	time averaged temperature at any point	K
T'	fluctuating component of temperature	K
$\bar{u}$	time averaged vertical velocity	m/s
u'	fluctuating component of vertical velocity	m/s
u^+	dimensionless vertical velocity, $\bar{u}/u_\tau$	
u_c	characteristic velocity, defined in Eq. (10)	
u_τ	frictional velocity, $\sqrt{\tau_w/\rho}$	m/s
u_∞	vertical velocity at inlet	m/s
$\bar{v}$	time averaged horizontal velocity	m/s
v'	fluctuating component of horizontal velocity	m/s
x	vertical co-ordinate	m
y	horizontal co-ordinate	m
y^+	dimensionless wall co-ordinate, yu_τ/ν	
$y^\times$	dimensionless wall co-ordinate for the inner layer, defined in Eq. (15)	

Greek symbols

α	thermal diffusivity	m^2/s
β	volumetric expansion coefficient	1/K
γ	blending parameter, defined in Eq. (12)	
$\Delta\overline{T}$	temperature difference, $T_H - T_C$	K
η	dimensionless wall distance, defined in Eq. (5)	
$\Theta^\times$	dimensionless near wall temperature, defined in Eq. (4)	
ρ	density	kg/m^3
τ_w	wall shear stress	N/m^2
τ_t^+	dimensionless shear stress, $\tau_t/\rho u_\tau^2$	
ν	kinematic viscosity	m^2/s
ψ	modified blending parameter	

numerically studied laminar-turbulent transition phenomena for buoyancy assisted and buoyancy opposed heated vertical channel flows during the early transient stage.

The above review of literature suggests that though several studies on turbulent mixed convection are reported in literature, systematic approaches for wall functions that can be applied to mixed convection are not that many. One such approach was advocated by Craft et al. [13], wherein a simple analytical approach was used to develop new wall functions. They also demonstrated the validity of their new wall functions by considering a few representative problems, one of which was mixed convection from a vertical pipe. However, the flow considered was forced convection dominant and hence one cannot assume that the approach will work in cases where natural convection dominates or is present alone. Consequent upon this, their validity in the true mixed convection regime is not clear. The above discussion leads us to the important question of "What is the true mixed convection regime?" This question can be satisfactorily answered from an asymptotic perspective.

In the asymptotic limit of $Re \to \infty$ and $Gr \to \infty$, when we define the Richardson number, Ri as Gr/Re^2, we can identify three cases in the sense of so-called distinguished limits:

(i) $Re \to \infty$ and $Gr \to \infty$, with $Ri \to 0$: Forced convection dominated.
(ii) $Re \to \infty$ and $Gr \to \infty$, with $Ri = O(1)$: Full mixed convection.
(iii) $Re \to \infty$ and $Gr \to \infty$, with $Ri \to \infty$: Natural convection dominated.

The first step involves the consideration of (i) and (iii) as perturbation problems. Then within (i) and (iii), mixed convection can be treated as a parameter perturbation problem for $Ri \to 0$ and $1/Ri \to 0$, respectively.

Notwithstanding an increase in the number of DNS studies, near wall treatment continues to be widely used in commercial packages and in house codes, for wall bounded turbulent flows. By definition, in a near wall treatment, analytical functions are used for velocity and temperature near the walls, and these are known as wall functions. The major advantage is that by using wall treatment there is tremendous saving on the computational resources and time, in view of the reduction in the number of grid points required to obtain stable, convergent and reliable solutions. However, wall functions were originally developed for simple forced convection flows and so they perform poorly in some flows like sudden expansion/contraction, those with adverse pressure gradient and buoyancy induced flows.

In what follows, we examine the possibility of blending the wall functions for natural and forced convection, so that the "blended wall function" can be used for all values of the Richardson number.

2. The blending procedure

As already mentioned, mixed convection has two limits: natural and forced convection. An approach to describe the temperature and velocity wall functions for mixed convection should also be capable of describing the two limits correctly. One possible approach to ensure this is to blend the profiles of the two limits. The question then is: *Which quantity should be used as a blending parameter*? The Richardson number is not a suitable parameter as it is a global quantity. A blending parameter, however, must be based on local quantities defined at the wall.

In view of the above reasons, a new parameter γ should be defined that is (i) similar to the Richardson number in the sense that it has to take into account buoyancy, viscous and inertial forces and (ii) not based on a global quantity like the Ri itself. In order to arrive at this parameter, we take recourse to the governing equations in the near wall region, for two dimensional turbulent flow, for a typical geometry like the infinite vertical channel shown in Fig. 1. These are:

$$0=\frac{\partial}{\partial y}\left(\nu\frac{\partial \bar{u}}{\partial y}-\overline{u'v'}\right)+g\beta(\overline{T}-T_o) \tag{1}$$

$$0=\frac{\partial}{\partial y}\left(\alpha\frac{\partial \overline{T}}{\partial y}-\overline{v'T'}\right)\Leftrightarrow$$

$$\alpha\frac{\partial \overline{T}}{\partial y}\bigg|_w=\alpha\frac{\partial \overline{T}}{\partial y}-\overline{v'T'}=-\frac{q_w}{\rho c_p}=\text{const} \tag{2}$$

Eq. (1) is the x momentum equation that includes the effect of buoyancy and Eq. (2) is the energy equation where q_w is the wall heat flux density. In both the equations, the left hand side is 0 signifying that the convection terms are set to 0, as is normally done in the analysis of fluid flows in the near wall region [14].

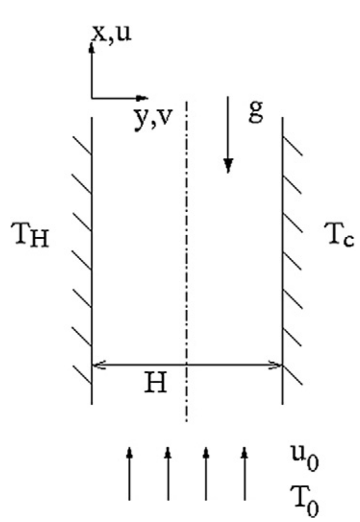

Fig. 1. Schematic of the infinite vertical channel geometry.

For the pure natural convection case, a characteristic temperature, T_c can be established using Buckingham's Π theorem as

$$T_c\equiv\left(\frac{\alpha^2}{g\beta}\left|\frac{\partial \overline{T}}{\partial y}\right|_w^3\right)^{1/4} \tag{3}$$

A dimensionless temperature can be defined as

$$\Theta^{\times}\equiv\frac{T_W-\overline{T}}{T_c} \tag{4}$$

In the above equation, T_W is the hot wall temperature. Similarly, a dimensionless wall co-ordinate can be introduced as:

$$\eta\equiv\frac{y}{H} \tag{5}$$

With the above definitions, Eq. (2) can be reduced to the following dimensionless form:

$$\frac{1}{(\frac{g\beta T_c H^3}{\alpha^2})^{1/3}}\frac{\partial \Theta^x}{\partial \eta}+q_{\text{turb}}^{+}=1 \tag{6}$$

where q_{turb}^{+} is the dimensionless turbulent heat flux density, given by $\overline{v'T'}/q_w$.

Eq. (6) can be recast as

$$\frac{1}{(Gr_t\,Pr^2)^{1/3}}\frac{\partial \Theta^x}{\partial \eta}+q_{\text{turb}}^{+}=1 \tag{7}$$

where Gr_t is the turbulent Grashof number given by $Gr_t=g\beta T_c H^3/\nu^2$. For pure forced convection flows, Eq. (1) can be reduced to the following dimensionless form [14].

$$\frac{1}{Re_t}\frac{\partial u^+}{\partial \eta}+\tau_t^+=1 \tag{8}$$

Where the dimensionless velocity $u^+=\bar{u}/u_\tau$, with u_τ being the frictional velocity given by $\sqrt{\tau_w/\rho}$. The dimensionless shear stress τ_t^+ is $\tau_t/\rho u_\tau^2$.

On comparing Eqs. (7) and (8) it is clear that the parameter $(\frac{Gr_t Pr^2}{Re_t^3})^{1/3}$ will be the appropriate blending parameter. One may call Gr_t/Re_t^3 as the turbulent Richardson number, Ri_t, and so the blending parameter, γ is

$$\gamma=\left(Ri_t Pr^2\right)^{1/3} \tag{9}$$

In natural convection flows the characteristic velocity u_c can be defined as

$$u_c\equiv\left(\frac{g\beta T_c^3}{\nu}\left|\frac{\partial \overline{T}}{\partial y}\right|_w^{-2}\right) \tag{10}$$

The above expression can also be derived using the Π theorem. With the above definitions

$$Ri_t=Pr^4(u_c/u_\tau)^3 \tag{11}$$

Substituting for Ri_t in Eq. (9), we have

$$\gamma=Pr^2\frac{u_c}{u_\tau} \tag{12}$$

In what follows, we look at the flows with vanishing but nonzero shear stress. Thus $u_\tau\neq 0$ and γ shows no singularity.

For the case of forced convection flows, $g=0$ and so $\gamma=0$. For the natural convection limit (at $Pr=0.7$) the ratio of the two

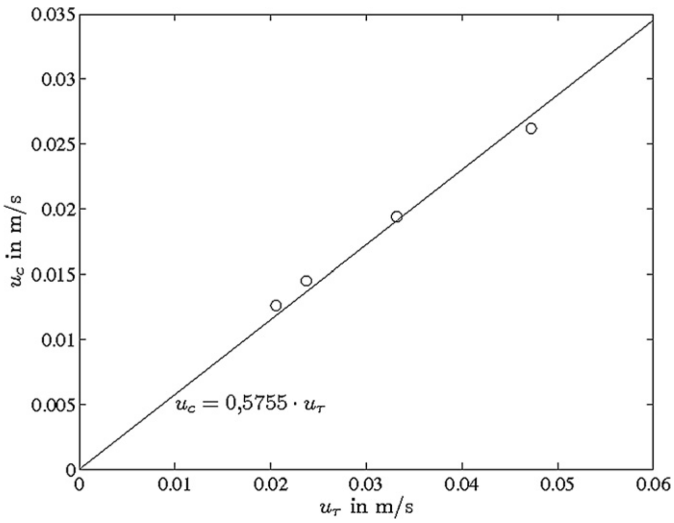

Fig. 2. Plot of u_c vs u_T for natural convection based on the data of Versteegh and Nieuwstadt [15].

characteristic velocities is approximately $u_c/u_T = 0.58$, when we use the DNS data of Versteegh and Niueuwstadt [15]. This can be seen in Fig. 2.

This results in the natural convection limit for γ_{nat} (at $Pr = 0.7$)

$$\gamma_{\text{nat}} = (0.7)^2(0.58) = 0.284 \tag{13}$$

Therefore, γ is characterized by

- $0 \leqslant \gamma \leqslant 0.284$ $(= \gamma_{\text{nat}})$.
- $\gamma = 0$: forced convection.
- $\gamma = 0.284$: natural convection.

Now, we have a blending parameter that is based on (local) wall quantities and stays between the two limits (forced and natural convection). With this parameter, a blending between the two limits should be possible. However, it would be more convenient to have a parameter that ranges between 0 and 1. This can be achieved with the following:

$$\psi \equiv \frac{\gamma}{0.284} \tag{14}$$

2.1. Transformation of the temperature profile

In this section, we look only at the temperature profile, though the procedure to be detailed out is equally applicable for the velocity profile, as well. The starting point is the temperature profile in the overlap layer of the near wall region that lies between the inner and outer layers. This can be derived from Eq. (2) using matched asymptotics (hereon the superscripts on the variables are removed for simplicity). The dimensionless wall coordinates for the near wall region for natural convection and forced convection are different. For natural convection, $y^\times$ is given by

$$y^\times \equiv \frac{y}{T_c}\left|\frac{\partial \overline{T}}{\partial y}\right|_w \tag{15}$$

While for forced convection, y^+ is given by $u_\tau y/\nu$.

A possible approach for blending is the transformation of the universal temperature profile for natural convection into the coordinates widely used in the analysis of forced convection flows, i.e. $y^\times \to y^+$ and $\Theta^\times \to \Theta^+$. The dimensionless near wall temperature for forced convection Θ^+ is given by $(T_W - T)/T_T$. We now formally perform the transformation, beginning with the near wall temperature profile for natural convection [16,17].

$$\frac{T_w - T}{T_c} = C \ln\left[\frac{y}{T_c}\left|\frac{\partial T}{\partial y}\right|_w\right] + D \tag{16}$$

where C and D are constants to be calibrated by matching the temperature profile with available DNS or experimental results. For a detailed derivation of Eq. (16), see [16,17].

$$\frac{T_w - T}{T_T}\left(\frac{T_T}{T_c}\right) = C \ln\left[\frac{y u_\tau}{\nu}\frac{\nu}{u_\tau T_c}\left|\frac{\partial T}{\partial y}\right|_w\right] + D \tag{17}$$

In the above equation, T_T is the frictional temperature given by

$$T_T = \frac{q_w}{\rho c_p u_\tau} \tag{18}$$

$$\Theta^+_{\text{nat}} Pr \frac{u_c}{u_T} = C \ln\left[y^+ Pr^2 \frac{u_c}{u_T}\right] + D \tag{19}$$

$$\Theta^+_{\text{nat}} = \frac{1}{Pr}\frac{u_T}{u_c}\left[C \ln(y^+) - C \ln\left(\frac{1}{Pr^2}\frac{u_T}{u_c}\right) + D\right] \tag{20}$$

In the above equation $u_T/u_c = 1/0.58$ can be used for the natural convection limit. The temperature profile for forced convection is known from literature [14].

$$\Theta^+_{\text{forced}} = \frac{1}{\kappa_\theta}\ln(y^+) + C^+(Pr) \tag{21}$$

Thus, a blending should be possible using the linear approach:

$$\Theta^+_{\text{mix}} = \Theta^+_{\text{forc}}(1-\psi) + \Theta^+_{\text{nat}}\psi \tag{22}$$

An alternative blending could be done with a potential approach:

$$\Theta^+_{\text{mix}} = \Theta^{+\,(1-\psi)}_{\text{forc}}\,\Theta^{+\,\psi}_{\text{nat}} \tag{23}$$

For $\psi = 1$ (natural convection) this reads as $\Theta^+_{\text{mix}} = \Theta^+_{\text{nat}}$ and for $\psi = 0$ (forced convection), $\Theta^+_{\text{mix}} = \Theta^+_{\text{forc}}$.

2.2. Validation of the blending approach

For validating the blending approach outlined in Section 2.1, the DNS data of Kasagi and Nishimura [8] for an infinite vertical plate channel, the same geometry that was used in [15], except for the fact that in [8] an additional feature is the uniform inlet velocity. In [8], calculations have been reported for one parameter set ($Re_T = 150$, $Gr = 9.6 \times 10^5$, $Re = 4494$, $Ri = 0.05$). It is evident that Re, Gr and Re_t are "low" in an asymptotic sense. Recently, Zanoun et al. [18] conducted high precision experiments to evaluate the law of the wall in two dimensional turbulent flows. They concluded that for low Reynolds numbers, i.e. $Re_t < 2 \times 10^3$, both the Kármán constant $(1/\kappa)$ and the additive constant (C^+) are not universal but are Reynolds number dependent (though in [18] only the

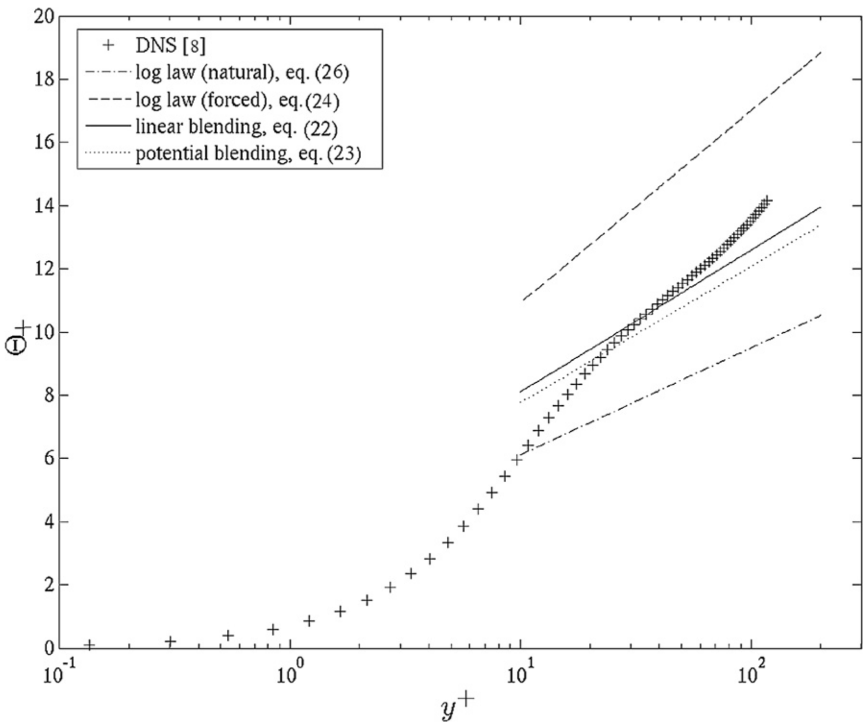

Fig. 3. Plot highlighting the blending procedure for the temperature profile in respect of the infinite channel.

velocity profile was discussed, the arguments hold for the temperature profiles as well). By the same token, for natural convection flows from [16] it is clear that the constants C and D are Rayleigh number dependent for Ra of the order of 10^5 and 10^6. Unfortunately, the Rayleigh number in [8] falls within this range.

In consideration of the above reasons, the blending has to proceed with caution. In Eq. (21), for Θ^+_{forc}, we evaluated the constants directly from the DNS data of [8] for the case of no buoyancy and the resulting temperature profile is:

$$\Theta^+_{\mathrm{forc}} = 2.64\ln(y^+) + 4.86 \tag{24}$$

For the same Prandtl number, the universal temperature profile for $Re \to \infty$ would have been [14]

$$\Theta^+_{\mathrm{forc}} = 2.13\ln(y^+) + 3.40 \tag{25}$$

The large difference between Eqs. (24) and (25) is a direct consequence of the low Reynolds number in the DNS case.

Similarly, the natural convection near wall temperature profile with the constants adjusted for a finite value of Ra, based on the recommendation given in [16], is

$$\Theta^+_{\mathrm{nat}} = 1.47\ln(y^+) + 2.73 \tag{26}$$

Using Eqs. (25) and (26), coupled with the ψ value for this parameter set (0.58), blending was done using both the linear and potential approaches as given in Eqs. (22) and (23) respectively and the results are seen in Fig. 3. The agreement between the blended profile and the data is reasonable. It is possible to make the agreement perfect by having a blending parameter that continuously changes with y^+. Shown in Fig. 4 is Fig. 3 replotted with a dynamically varying blending parameter. The agreement between the blended and the DNS profiles is now remarkable.

Table 1
Blending parameter for temperature wall functions for mixed convection from a vertical, parallel plate channel

S. No.	$Gr * 10^{-5}$	Re	Ri	Modified blending parameter, ψ_a
(a) Aiding flow				
1	6.4	4341	0.0339	0.295
2	9.6	4328	0.051	0.125
3	16	4148	0.093	0.09
(b) Opposing flow				
S. No.	$Gr * 10^{-5}$	Re	Ri	Modified blending parameter, ψ_0
1	6.4	4341	0.0339	0.651
2	9.6	4328	0.051	0.615
3	16	4148	0.093	0.605

However, it needs to be mentioned that by doing this, the simplicity that we are actually seeking in the blending procedure is more or less lost. Furthermore, generalization becomes more difficult. An important feature of the DNS profile is that the logarithmic portion of the curve does not span across a larger distance. This, again, is due to the low value of Re_T.

For further validation, five more cases of the same problem (2 aiding and 3 opposing flow) were considered with linear blending. Details of Gr, Re and Ri are given in Table 1. Using an exactly similar procedure, we obtained blended profiles for the temperature wall function for mixed convection. These were compared with the DNS results reported again in [8]. Fig. 5

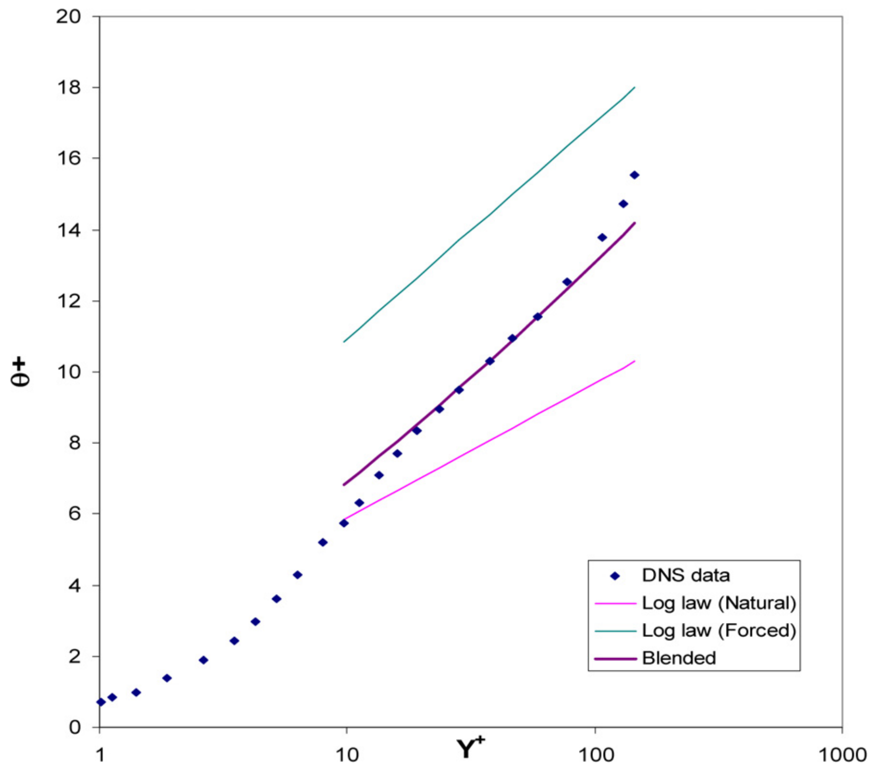

Fig. 4. Plot showing the near wall temperature profile for the vertical channel with a dynamically varying blending parameter. Logarithmic laws for natural convection, forced convection, the blended profile and the DNS data are all shown in the figure.

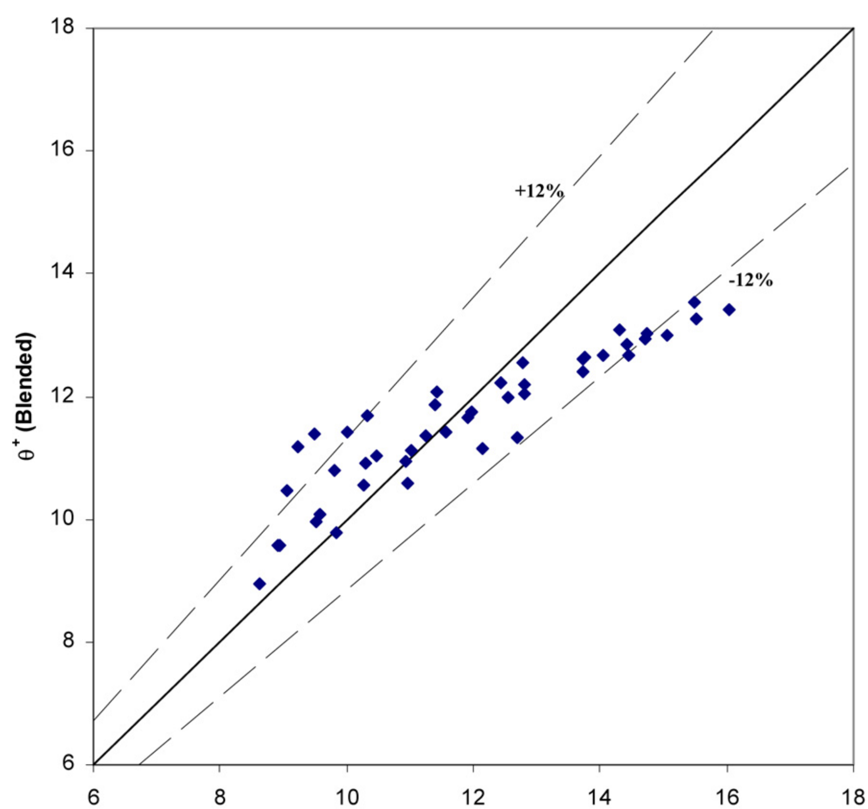

Fig. 5. Parity plot highlighting the adequacy of the blending procedure (points taken for three values of *Ri* for both aiding and opposing flows).

shows a parity plot of θ^+ (blended) vs θ^+ (DNS) for these cases. It can be seen that more than 90% of the points lie within the $\pm 12\%$ error band. Hence, for the geometry under consideration, this reinforces our confidence in the blending procedure.

3. Discussion

The treatment of mixed convection as a parameter perturbation problem with $\gamma = Pr^2 \frac{u_c}{u_T}$ for the blending of the wall function discussed in this study, is useful in deciphering the fundamental nature of the important quantities like the near wall temperature and velocity profile. Also, for complex geometries a "mixed convection wall function" will be of much help in reducing the computational requirements for carrying out a numerical investigation. The real problem here, however, is the lack of DNS and experimental results for further validation, that will add more confidence to the approach. Availability of reliable data will also help us evaluate whether the approach advocated in this study can be extended to such complex flows. Also, since buoyancy depends on the inclination of the surface losing heat, obtaining a universal function that is valid for all angles will be quite formidable.

We now look at the possibility of estimating the Nusselt number directly from the temperature wall function. In principle, this should be doable as the wall function does have information on the temperature gradient and hence the Nusselt number. This exercise is straightforward for fully developed flows [19], like natural convection from a parallel plate channel, where the temperature at the centre may be taken to be $(T_{\mathrm{left}} + T_{\mathrm{right}})/2$. Knowledge of temperature at one location (at least) in the region where the logarithmic law of the wall is valid is essential, so that the temperature wall function can be transformed into a Nusselt number correlation. The problem with mixed convection is that the centerline temperature will not be 0.5 and will be a function of Ri. Therefore, the transformation of the wall function to Nusselt number is quite tedious. Even so, with the DNS data of [8] and the blended wall function proposed in this paper, the difference in Nusselt numbers as predicted by the wall function and that obtained directly by DNS for the case of aiding flow when $Re = 4328$, $Gr = 9.6 \times 10^5$ and $Ri = 0.05$ was around 10%. Hence, conceptually one can estimate the Nusselt number from the wall function though for mixed convection the procedure is quite involved.

4. Conclusions and outlook

Using asymptotic considerations, the treatment of turbulent mixed convection as a parameter perturbation problem in Richardson number was demonstrated. Temperature wall functions for natural and forced convection were suitably blended in order to arrive at a wall function for mixed convection. The use of wall functions represents a computationally efficient method of handling turbulent mixed convection. More importantly, asymptotic considerations provide them the credibility and the much needed physical basis for deriving them.

The analysis also clearly brings out the need for more DNS results for mixed convection, so that we can have more faith in the methodology presented here. Future studies may also consider obtaining the wall functions for mixed convection from the Nusselt number (or skin friction) data itself obtained either through DNS or experiments.

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